

Quadratic Chaotic Inflation from Higgs Inflation

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Abstract

Stimulated with the recent discovery of B-mode by BICEP2, we discuss the relation between a Higgs inflation and a chaotic inflation with quadratic potential. Starting with a generalized Higgs inflation model, we derive a condition for obtaining the quadratic chaotic inflation. It is shown that the running of the Higgs self-coupling constant in the Jordan frame plays a decisive role when the generalized Higgs inflation model coincides with the Higgs inflation model in a small-field limit.

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1 Introduction

The BICEP2 experiment has recently announced a remarkable discovery of the primordial B-mode polarization in the cosmic microwave background (CMB) [1], thereby giving us a strong support for the inflation scenario [2]. According to the BICEP2 results [1], the tensor-to-scalar ratio is given by $r = 0.20^{+0.07}_{-0.05}$ (this value is decreased to $r = 0.16^{+0.06}_{-0.05}$ after subtracting the best available estimate for foreground dust), which suggests that a large-field inflation has happened at a very early stage of evolution of the universe. Then, inflation is sensitive to quantum gravity and string theory, so for the first time we might have a chance of understanding quantum gravitational effects experimentally by examining the CMB carefully.

Of particular interest is that the BICEP2 results favor the simplest chaotic inflation [3] with quadratic potential

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2, \quad (1)$$

where ϕ is an inflaton field and m is its mass. An important issue here is the identification of the inflaton field ϕ .

As the most economical approach, it is tempting to identify the inflaton field ϕ with the Higgs field in the standard model (SM), which has been found at the LHC recently [4, 5]. Indeed, such an inflation model is nowadays called the Higgs inflation which has been actively investigated so far [6]. A peculiar feature of the Higgs inflation is the presence of a non-minimal coupling term of the inflaton to gravity, by which the flatness of the potential at large values of the inflaton is ensured. Once the non-minimal coupling constant is experimentally fixed by the amplitude of the scalar perturbations, the theory has a strong predictive power. For instance, this model predicts the spectral index $n_s \approx 0.97$ and the tensor-to-scalar ratio $r \approx 0.003$. It is worthwhile to recall that one of features in the Higgs inflation is a lowering of the tensor-to-scalar ratio. Actually, the prediction of the tensor-to-scalar ratio $r \approx 0.003$ by the Higgs inflation is obviously incompatible with the larger value $r \approx 0.20$ by the BICEP2, which yielded a conjecture that the Higgs inflation might be dead [7].

However, more recently, the paper entitled "Higgs inflation still alive" has appeared where a critical observation is that by tuning of the top quark mass one can make a saddle point in the Higgs potential [8].³ Then, it is pointed out that the e-folding is earned by passing the saddle point, which leads to the almost same tensor-to-scalar ratio as that by BICEP2, and the observational density perturbation corresponds to the size of the Higgs field above the saddle point.

In this article, we wish to clarify the relation between the Higgs inflation and the chaotic inflation with quadratic potential. The key point is that we must make use of not the constant but the running Higgs self-coupling constant with a minimum near the Planck mass scale [10]. Although there are two prescriptions identifying the renormalization scale using the Einstein frame and the Jordan one, leading to the same conclusion in Ref. [8], our formalism prefers the Jordan frame to the Einstein one.

³See also a related work [9].

This article is organized as follows: In the next section, we construct a generalized Higgs inflation in the Jordan frame which agrees with the conventional Higgs inflation at small values of the Higgs field. In Section 3, we derive an equation between the generic functions by requiring the generalized Higgs inflation to become equivalent to the quadratic chaotic inflation in the Einstein frame. In Section 4, we consider the case of the original Higgs inflation and show that the running of the Higgs self-coupling constant in the Jordan frame plays a decisive role when the generalized Higgs inflation model coincides with the Higgs inflation model. The final section is devoted to discussion.

2 Generalized Higgs inflation

Let us begin with the construction of the generalized Higgs inflation where compared to the original Higgs inflation [6] the factors in front of both the scalar curvature and the Higgs kinetic term are extended to generic functions of the Higgs field.⁴ A portion of the SM Lagrangian relevant to our argument takes the following form:

$$\mathcal{L} = \sqrt{-g} \left[F(H)R - K(H)g^{\mu\nu}\partial_\mu H^\dagger\partial_\nu H - U(H) \right], \quad (2)$$

where H is the Higgs doublet for which we take the unitary gauge $H = \frac{1}{\sqrt{2}}(0, h)^T$ throughout this article. Moreover, $U(H)$ is the conventional Higgs potential defined as

$$U(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 = \frac{\lambda}{4} (h^2 - v^2)^2 \equiv U(h), \quad (3)$$

with λ and v being the Higgs self-coupling constant and the vacuum expectation value of the Higgs field, respectively.

In order to move from the Jordan frame to the Einstein frame, let us perform the "conformal" transformation⁵

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x)g_{\mu\nu}, \quad g^{\mu\nu} \rightarrow \tilde{g}^{\mu\nu} = \Omega^{-2}(x)g^{\mu\nu}, \quad (4)$$

from which we can derive useful formulae:

$$\sqrt{-g} = \Omega^{-4}\sqrt{-\tilde{g}}, \quad R = \Omega^2(\tilde{R} + 6\Box f - 6\tilde{g}^{\mu\nu}\partial_\mu f\partial_\nu f), \quad (5)$$

where we have defined as $f = \log \Omega$ and $\Box f = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\tilde{g}^{\mu\nu}\partial_\nu f) = \tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu f$.

With the choice of the scale factor Ω satisfying⁶

$$\Omega^2 = \frac{2F(h)}{M_p^2}, \quad (6)$$

⁴A generalized Higgs inflation based on the Galieon field has been considered in [11].

⁵Precisely speaking, this transformation is not the conformal one since the Higgs field remains intact [12].

⁶Here $M_p = \sqrt{\frac{c\hbar}{8\pi G}} = 2.44 \times 10^{18} GeV$ is the reduced Planck mass.

it is easy to rewrite the Lagrangian (2) up to a surface term as

$$\mathcal{L} = \sqrt{-\tilde{g}} \left\{ \frac{1}{2} M_p^2 \tilde{R} - \frac{M_p^2}{4} \left[\frac{K}{F} + 3 \left(\frac{F'}{F} \right)^2 \right] \tilde{g}^{\mu\nu} \partial_\mu h \partial_\nu h - V(h) \right\}, \quad (7)$$

where prime means the derivative with respect to h , i.e., $F' = \frac{dF}{dh}$ and the new potential in the Einstein frame is defined as

$$V(h) = \frac{\lambda M_p^4}{16} \left(\frac{h^2 - v^2}{F(h)} \right)^2. \quad (8)$$

Provided that one introduces a new scalar field χ defined as

$$\frac{d\chi}{dh} = \frac{M_p}{\sqrt{2}} \frac{\sqrt{K(h)F(h) + 3(F'(h))^2}}{F(h)}, \quad (9)$$

the Lagrangian (7) can be cast to

$$\mathcal{L} = \sqrt{-\tilde{g}} \left[\frac{1}{2} M_p^2 \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(h(\chi)) \right]. \quad (10)$$

This is the Lagrangian of our generalized Higgs inflation in the Einstein frame. We will require this Lagrangian to become the quadratic chaotic inflation by restricting the form of the potential $V(h(\chi))$.

3 Quadratic chaotic inflation

Now we wish to have a chaotic inflation with quadratic potential from the above generalized Higgs inflation (10). To this aim, the new potential $V(h(\chi))$ must take the form

$$V(h(\chi)) = \frac{1}{2} m^2 \chi^2, \quad (11)$$

where m is the mass of the scalar field χ . Together with Eqs. (8) and (11), χ is expressed in terms of h as

$$\chi = \frac{M_p^2}{2m} \sqrt{\frac{\lambda}{2}} \frac{h^2 - v^2}{F(h)}. \quad (12)$$

It is known that the Higgs self-coupling constant λ has a peculiar behaviour as a function of the renormalization scale μ . For instance, around the Planck mass scale M_p , the running coupling constant $\lambda(\mu)$ has a minimum at $\mu_{min} \approx 10^{17-18} GeV$ depending on the Higgs mass and can be approximated as

$$\lambda(\mu) = \lambda_{min} + \frac{b}{(16\pi^2)^2} \left(\log \frac{\mu}{\mu_{min}} \right)^2, \quad (13)$$

where $b \approx 0.6$ [8, 9, 13]. There are two prescriptions how to select the renormalization scale μ . One prescription is to choose the renormalization scale μ to be the effective mass in the Einstein frame

$$\mu = \frac{M_p}{\sqrt{2F(h)}}h. \quad (14)$$

The other prescription is to choose it to be the effective mass in the Jordan frame

$$\mu = h. \quad (15)$$

Even if both the prescriptions work well in Ref. [8], we have to make use of the latter prescription in this study as seen shortly. With this prescription, the running coupling constant becomes a function of h as

$$\lambda(\mu) = \lambda(h). \quad (16)$$

Finally, let us note that the mathematical consistency requires the differentiation of χ in Eq. (12) with respect to h to be equal to Eq. (9). Using this consistency condition, we can express the function $K(h)$ like

$$K = \frac{\lambda M_p^2}{4m^2} \frac{1}{F} \left[\frac{1}{2} \frac{\lambda'}{\lambda} (h^2 - v^2) + 2h - (h^2 - v^2) \frac{F'}{F} \right]^2 - 3 \frac{(F')^2}{F}. \quad (17)$$

4 Higgs inflation

Now let us consider the Higgs inflation in the Jordan frame by specifying the function $F(h)$ to be the following form [6]⁷

$$F(h) = \frac{1}{2}(M_p^2 + \xi h^2). \quad (18)$$

With the choice (18), the function $K(h)$ in Eq. (17) is given by

$$K = \frac{\lambda M_p^2}{2m^2(M_p^2 + \xi h^2)} \left[\frac{1}{2} \frac{\lambda'}{\lambda} (h^2 - v^2) + \frac{2(M_p^2 + \xi v^2)}{M_p^2 + \xi h^2} h \right]^2 - \frac{6\xi^2 h^2}{M_p^2 + \xi h^2}. \quad (19)$$

Since the running coupling constant λ is a complicated and numerical function, we shall confine ourselves to the analysis of the function $K(h)$ only in the limits of both $\xi h^2 \gg M_p^2$

⁷Although it is possible to generalize the function $F(h)$ to a more general expression such as $F(h) = \frac{1}{2}M_p^2 \left(1 + \frac{\xi h^2}{nM_p^2}\right)^n$ which reduces to (18) in the limit $\xi h^2 \ll M_p^2$, it turns out that we obtain a similar result to that in case of (18).

and $\xi h^2 \ll M_p^2$. In the limit $\xi h^2 \gg M_p^2$, since λ is approximated as in Eq. (13), K is reduced to

$$K \rightarrow k_1 \equiv \frac{b}{2(16\pi)^2 \xi} \left(\frac{M_p}{m} \right)^2 - 6\xi. \quad (20)$$

The requirement that the Higgs particle is not a ghost but a normal particle imposes the constraint that k_1 in Eq. (20) should be positive definite. Then, the non-minimal coupling constant ξ is bounded by

$$\xi < \frac{1}{32\pi} \sqrt{\frac{b}{3}} \frac{M_p}{m} \approx 7 \times 10^2, \quad (21)$$

where we have put $b = 0.6$, $m = 1.5 \times 10^{13} \text{GeV}$, $M_p = 2.44 \times 10^{18} \text{GeV}$. This inequality certainly covers the value $\xi \approx 10$ in Ref. [8]. With the redefinitions $\sqrt{k_1}h = \bar{h}$, $\sqrt{k_1}v = \bar{v}$, $\frac{1}{k_1^2}\lambda = \bar{\lambda}$ and $\frac{1}{k_1}\xi = \bar{\xi}$, the starting Lagrangian (2) can be rewritten in the limit $\xi h^2 \gg M_p^2$ as

$$\mathcal{L} \approx \sqrt{-g} \left[\frac{1}{2} (M_p^2 + \bar{\xi} \bar{h}^2) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \bar{h} \partial_\nu \bar{h} - \frac{\bar{\lambda}}{4} (\bar{h}^2 - \bar{v}^2)^2 \right], \quad (22)$$

which is nothing but the Lagrangian of the Higgs inflation. It is of interest to note that the Lagrangian of the generalized Higgs inflation takes the same form as that of the Higgs inflation even in the region of large field.

Next we turn our attention to the opposite limit $\xi h^2 \ll M_p^2$. In this limit, the Lagrangian (2) must become that of the Higgs inflation in order to ensure that our model in fact describes the Higgs inflation. The running coupling constant in this limit is approximated as

$$\lambda(h) = \lambda_0 e^{-\alpha h}, \quad (23)$$

where λ_0 and α are positive constants. Then, K reads

$$K \rightarrow k_2 \equiv 2\lambda_0 \left(\frac{v}{m} \right)^2, \quad (24)$$

which is positive definite as desired.⁸ Thus, with the redefinitions $\sqrt{k_2}h = \hat{h}$, $\sqrt{k_2}v = \hat{v}$, $\frac{1}{k_2^2}\lambda = \hat{\lambda}$ and $\frac{1}{k_2}\xi = \hat{\xi}$, it is certainly true that in the limit $\xi h^2 \ll M_p^2$ the Lagrangian (2)

⁸This result is robust against changes in the approximation of the running coupling constant. For instance, if we approximate the running coupling constant in this limit by using a quadratic function as

$$\lambda(h) = \lambda_0 (h - h_0)^2, \quad (25)$$

where λ_0 and h_0 are some positive constants (this equation holds for $h < h_0$), K is given by

$$K \rightarrow k_2 \equiv 2\lambda_0 \left[\frac{(v - h_0)v}{m} \right]^2, \quad (26)$$

which is positive definite as well.

can be cast to be the same form as that of the Higgs inflation

$$\mathcal{L} \approx \sqrt{-g} \left[\frac{1}{2} (M_p^2 + \xi \hat{h}^2) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \hat{h} \partial_\nu \hat{h} - \frac{\hat{\lambda}}{4} (\hat{h}^2 - \hat{v}^2)^2 \right]. \quad (27)$$

5 Discussion

In this article, we have clarified the relation between the Higgs inflation and the quadratic chaotic inflation when the running of the Higgs self-coupling constant is switched on. Via the construction of the generalized Higgs inflation model, we have shown that the Higgs inflation in the Jordan frame can be described by quadratic chaotic inflation in the Einstein frame in treating with both the running coupling constant and the kinetic term in a proper manner.

Moreover, it is shown that within the present framework, the Lagrangian of the generalized Higgs inflation is reduced to that of the usual Higgs inflation in both large and small field limits. To do that, it is essential to choose the renormalization scale μ to be the Higgs field h , which is done only in the Jordan frame. In this sense, in the formalism at hand, the prescription in the Jordan frame possesses a more preferred position than that in the Einstein frame.

From our work, it is found that the Higgs inflation with the running of the Higgs self-coupling constant and the generalized Higgs kinetic term in the Jordan frame can be precisely rewritten as a quadratic chaotic inflation in the Einstein frame, which nicely explains the recent results by BICEP2 [1]. Usually, in the Higgs inflation to account for the BICEP2 results [8, 9], even if the running of the Higgs self-coupling constant is taken into consideration, the kinetic term of the Higgs field is kept to be canonical, so it is difficult to rewrite the Lagrangian in the Jordan frame to that of the quadratic chaotic inflation in the Einstein frame. Thus, our study might shed a light on the importance of the modification of the Higgs kinetic term in regarding the Higgs field as the inflaton.

As a future work, we wish to report the unitarity issue [14] in the present formalism.

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References

- [1] P. A. R. Ade et al. [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].
- [2] A. A. Starobinsky, JETP Lett. **30** (1979) 682 [Pis'ma Zh. Fiz. **30** (1979) 719]; K. Sato, Mon. Not. Roy. Astron. Soc. **195** (1981) 467; A. H. Guth, Phys. Rev. **D 23** (1981) 347.
- [3] A. D. Linde, Phys. Lett. **B 129** (1983) 177.

- [4] G. Aad et al. [ATLAS Collaboration], Phys. Lett. **B 716** (2012) 1, arXiv:1207.7214 [hep-ex].
- [5] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. **B 716** (2012) 30, arXiv:1207.7235 [hep-ex].
- [6] F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. **B 659** (2008) 703, arXiv:0710.3755 [hep-th]; F. L. Bezrukov, Class. Quant. Grav. **30** (2013) 214001, arXiv:1307.0708 [hep-ph] and references therein.
- [7] J. L. Cook, L. M. Krauss, A. J. Long and S. Sabharwal, arXiv:1403.4971 [astro-ph.CO].
- [8] Y. Hamada, H. Kawai, K.-y Oda and S. C. Park, arXiv:1403.5043 [hep-ph].
- [9] F. L. Bezrukov and M. Shaposhnikov, arXiv:1403.6078 [hep-ph].
- [10] Y. Hamada, H. Kawai and K.-y Oda, PTEP **2014** (2014) 023B02, arXiv:1308.6651 [hep-ph].
- [11] I. Oda, Phys. Rev. **D 87** (2013) 065025, arXiv:1301.2709 [hep-ph]; I. Oda, Phys. Lett. **B 724** (2013) 160, arXiv:1305.0884 [hep-ph]; I. Oda, Adv. Studies in Theor. Phys. **8** (2014) 215, arXiv:1308.4428 [hep-ph].
- [12] K. Kamada, T. Kobayashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. **D 83** (2011) 083515, arXiv:1012.4238 [astro-ph.CO]; K. Kamada, T. Kobayashi, T. Takahashi, M. Yamaguchi and J. Yokoyama, Phys. Rev. **D 86** (2012) 023504, arXiv:1203.4059 [hep-ph]; K. Kamada, T. Kobayashi, T. Kunimitsu, M. Yamaguchi and J. Yokoyama, Phys. Rev. **D 88** (2013) 123518, arXiv:1309.7410 [hep-ph].
- [13] A. Salvio and A. Strumia, arXiv:1403.4226 [hep-ph].
- [14] H. M. Lee, arXiv:1403.5602 [hep-ph].